Computational and Experimental Mathematics: Introduction

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Overview

This session will have two components

- A hands on introduction to SAGE,
  a new and impressive computational tool for math
- An exploration of some experimental mathematics

This introductory session will be accessible to math aficionados at all levels.

Setting up a Sage session

To get started with Sage open a browser, preferably Firefox, and

- Surf to the Sage Server at Clemson
  `https://clemix.clemson.edu:34567/`

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- Click on Published
- Select the worksheet `SC10CE1a`
- Click on “Edit a copy”

The Sage Intro 1 Worksheet

- The Sage notebook uses blocks of text
  essentially Python statements that get executed
- Lines that start with the pound sign are comments
- Blocks with HTML and LaTeX formatting are also possible
- A block is evaluated by clicking on the evaluate link
  or by typing shift-enter

Sage and Elementary Calculations

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- Sage's graphics capabilities can provide helpful illustrations
- Sage can also do Calculus

Twentieth Century Results

Substantial Results from the last Century radically changed our understanding of the Mathematical Process

- The Century started with Hilbert's 23 problems.
- 10 clear solutions
  for several the answer it can't be done
- 8 controversial including Kepler Conjecture
  computer assisted - not human verifiable
- 2 clearly unresolved
- 4 too vague to solve

Twentieth Century Results

Additional radical changes include

- Gödel's Incompleteness Theorem
- The computer and the Four Color Theorem
- The Classification of the Simple Groups
  5000 pages of Journal articles and there is no 'ogre'

Experimental Mathematics

Some ways computation can help

- computing for insights (to compute about something you have to understand how it fits together)
- discovering new facts, patterns, and relationships
- visualizing to expose facts, structures, and principles
- rigorously testing and falsifying conjectures
- survey the landscape
- suggesting approaches for formal proof using symbolic computation
- replacing lengthy hand derivations with computation
- confirming analytically derived results

Set up the Second Intro Worksheet

Save and quit the first worksheet or

- Surf to the Sage Server at Clemson and login `https://clemix.clemson.edu:34567/`
- Click on Published
- Select the worksheet `SC10CE1b`
- Click on “Edit a copy”

Experimental Math Worksheet

- Arithmetic progressions, Differences, and Sums

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- Pascal's Triangle and Diagonal Sums

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- Pascal's Triangle and Diagonal Sums
- Inverse Symbolic Calculator
  `http://oldweb.cecm.sfu.ca/projects/ISC/`

Our Thanks

Our thanks to

- The SuperComputing 2010 Conference, the organizers of the educational program, and all the affiliated sponsors.

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- You, the audience, for your attention